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Branches of the landscape

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Abstract: With respect to the question of supersymmetry breaking, there are three branches of the flux landscape. On one of these, if one requires small cosmological constant, supersymmetry breaking is predominantly at the fundamental scale; on another, the distribution is roughly flat on a logarithmic scale; on the third, the preponderance of vacua are at very low scale. A priori, as we will explain, one can say little about the first branch. The vast majority of these states are not accessible even to crude, approximate analysis. On the other two branches one can hope to do better. But as a result of the lack of access to branch one, and our poor understanding of cosmology, we can at best conjecture about whether string theory predicts low energy supersymmetry or not. If we hypothesize that are on branch two or three, distinctive predictions may be possible. We comment of the status of naturalness within the landscape, deriving, for example, the statistics of the first branch from simple effective field theory reasoning.

Keywords: Superstring Vacua.

Contents

1. Introduction: landscape phenomenology

The cosmological constant problem is one of the greatest puzzles confronting particle theory and theories of gravity. Until very recently, string theory seemed to offer no solution to this problem. Now, however, there is evidence that the vast array of possible fluxes in string theory leads to a discretuum of metastable states [1–8]. The number may well be large enough that a non-trivial subset have cosmological constant of the order observed.

This strongly suggests that string theory can accommodate the observed cosmological constant. While one might hope that some cosmological effect would pick out the observed vacuum uniquely, it seems more likely that explaining the cosmological constant in such a framework will require invoking anthropic considerations. Absent any real understanding of string cosmology, adopting the hypothesis that somehow the universe, in its history, explores a large fraction of these states, in a more or less democratic fashion, predicts a cosmological constant in roughly the right range [9–11].

The question is: having adopted the hypotheses that the landscape exists and that the universe samples all of these states, can we explain or predict anything further? Doing so requires an understanding of the statistics of string vacuum states [12–16]. There has already been significant progress in this direction, and we can begin to address certain questions. Perhaps the most obvious is the origin of the hierarchy between the weak scale and the Planck (or other large) scales. The familiar proposals: low energy supersymmetry, with some sort of dynamical supersymmetry breaking, and technicolor have realizations in the landscape which are at least partially understood. So it is natural to ask whether the landscape might predict low energy supersymmetry, perhaps with some specific breaking scale and pattern of soft masses, or something like technicolor (or, probably equivalently, warped geometry), or whether something totally different might arise. Perhaps the most troubling alternative suggested by the landscape is that the solution is similar to that of the cosmological constant — there is simply a huge number of states, and anthropic considerations pick out the observed hierarchy, with no other low energy consequences.

In [17], a critique of this program was offered, and the very existence of the landscape was questioned (a more extensive discussion appears in [18], and some possible counter-arguments in [19]), but it was also suggested that if the landscape does exist, low energy supersymmetry would be a plausible outcome. This statement was met by some skepticism [20, 21]; it was argued that supersymmetry would necessarily be broken at the Planck scale. These arguments, while also plausible, relied on assumptions whose validity has not been established. A more refined description of how low energy supersymmetry might emerge was presented in [22]. These authors distinguished three branches of the landscape. In the case of type-IIB theories compactified on orientifolds of Calabi-Yau spaces (or F theory compactifications on Calabi-Yau four-folds), these correspond to:

1. States with broken supersymmetry at tree level
2. States with unbroken supersymmetry and $W \neq 0$ at tree level
3. States with unbroken supersymmetry and $W = 0$ at tree level.

An extensive statistical analysis of branch 2 was performed in [15]. The statistics of some simple models on branch three has been studied recently in [23]. More recently, in [16] the statistics of states on branch one with small scale of supersymmetry breaking has recently been analyzed.

In this paper, we will attempt to understand the statistics and the physics of these three branches further. We begin by noting that, non-perturbatively, there is no sharp distinction between the states on the different branches: superpotentials are likely generated and supersymmetry broken on both branches two and three. The real distinction between them lies in their statistics. If one considers the distribution of low energy lagrangians, on branch one the measure is smooth in the regions where F, the supersymmetry breaking scale and Λ are small. On branch two, it is mildly singular in these regions; on branch three it is very singular.

We will discuss a number of features of branch one. We review why, in the classical approximation, the number of stationary points of the supergravity potential is infinite, and discuss the possible physics of the cutoff. We give a simple, low energy effective action argument which yields the statistics of [16]. We explain why, on this branch, the vast majority of the states are not accessible to analysis, so that it is not possible to determine their statistics. This means that, at present, one cannot decide, by a priori theoretical arguments, whether this branch dominates, and, if this branch dominates, one cannot make predictions. We will give some reasons to conjecture that if this branch dominates, it

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leads to phenomena similar either to technicolor theories or to Randall Sundrum models. We argue that there may already be phenomenological considerations which rule out this possibility.

We then turn to branches two and three. We want to ask: if one adopts the hypothesis that the universe is not on branch one, can one make predictions for quantities such as the scale of supersymmetry breaking, and the pattern of soft breakings. We will review some of what is known about branch two, and discuss the question of computability. Ref. [23] considered models on branch three with small numbers of moduli. Another perspective on R symmetries was provided in [24]. We discuss the general problem of finding models with vanishing $\langle W \rangle$, arguing that this is most likely connected with discrete R symmetries. We explain that with discrete R symmetries statistical considerations will generally lead to supersymmetric ground states with vanishing W. We discuss in a well-known model the statistical cost of such discrete symmetries and find that it is substantial. But we note that a number of selection effects may still favor such vacua; moreover, as suggested in [23], the statistical cost may not always be so high.

We conclude with our views of the prospects for prediction within the landscape. The question of whether the theory predicts low energy supersymmetry, technicolor-like theories, or nothing beyond the standard model Higgs seems to be an experimental one, though there is some reason to think that the usual naturalness arguments — even in the landscape and even allowing possible anthropic considerations — will disfavor the last possibility. On the supersymmetric branches, there is good reason to think that low energy supersymmetry is favored. Determining whether there are more detailed predictions — gauge mediation, anomaly mediation, or something completely different, requires studies of the statistics of gauge groups beyond those which have been done to date, but which may well be feasible.

2. The statistics of branch one

In string theory, we have become accustomed to the idea that the more supersymmetry, the more control one has over the analysis of a problem. As we will see, many features of branch three are particularly easy to understand. Ref. [15] studies the statistics of branch two and some aspects of those of branch one. Like KKLT, these authors study principally the type IIB theory on an orientifold of a Calabi-Yau space. In this case, there is a superpotential for the complex structure moduli in the supergravity approximation, but not for the Kähler moduli. Noting KKLT's observation that the Kähler moduli are likely to be fixed by non-perturbative effects, they make the further simplification of ignoring the Kähler moduli. The most primitive aspect of statistics is to simply count the number of stationary points of the potential. Here already there is a striking difference between branches one and two; on the supersymmetric branch, the number of stationary points is finite, while on the non-supersymmetric branch, the number is divergent and it is necessary to introduce a cutoff.

In the limit in which the flux numbers are large, DD convert sums over fluxes to integrals over the moduli space. The result has the form, in the supersymmetric case (branch two):

$$\mathcal{N}_{\rm susy} = \frac{L^{K/2}}{(K/2)!}\mathcal{N}_o\,. \tag{2.1}$$

$\mathcal{N}_o$ is given by an expression of the form:

$$\mathcal{N}_o = \int d^{2m}z \int d^K N e^{-|X|^2-|Z|^2+|Y_i|^2}\delta^{2m}(DW)|D^2W|\,. \tag{2.2}$$

where

$$X = W \qquad Y_i = D_{z_i}W = F_{z_i}\,. \tag{2.3}$$

In the non-supersymmetric case, the term $\delta^{2m}(DW)$ is replaced by $\delta^{2m}(V')$, and an appropriate determinant We will not need the definition of Z; the main point is that it gives a negative contribution to the exponential. For supersymmetric vacua, Y vanishes, and the integral is finite. This is not the case for non-supersymmetric vacua, and Douglas and Denef argue that the integral is divergent, and impose a cutoff on the supersymmetry breaking scale ($F = Y = D_zW$), and argue that it should not be too large.

While we believe that there may well be a physical cutoff, we do not have a convincing argument as to whether a physical cutoff exists. Certainly one cannot calculate, even crudely, at large F; as we will explain, the α' expansion breaks down badly once F is comparable to the string scale. We will also see that, for F comparable to the string scale, most of the would-be metastable states are highly unstable. But while these arguments suggest that many would-be vacua are not meaningful states of the theory, some number of states — possibly infinite – may still exist. If there is a cutoff, there is no parameter which would make it small. So, even if we demand that states have small cosmological constant and are highly metastable, most such states will be at the string scale. The original assumption that IIB orientifolds (or F theory on CY fourfolds) are representative is thus not correct (or at least not meaningful) on this branch. The typical states do not look like anything we know how to describe. This means that we do not know how to do even the crudest counting on this branch.

2.1 An infinite number of non-supersymmetric states

We can demonstrate that the number of non-supersymmetric stationary points of the supergravity potential is infinite in a particular class of flux models: the IIB theory on T_6/Z_2 [25] The tadpole cancellation condition reads:

$$-a^o d_o - a^{ij}d_{ij} + b_{ij}c^{ij} + b_o c^o = \ell^* \tag{2.4}$$

for a fixed, positive integer ℓ^*. We can satisfy this condition by choosing:

$$b_o = -3N\,; \qquad c^o = N\,; \qquad c^{ij} = N\delta^{ij} = b_{ij} \tag{2.5}$$

and a^{ij}, a_o, d_o, d_{ij} numbers of order one which satisfy the constraint. Now N can be taken arbitrarily large, so it would appear that we have an infinite number of possible states. We might expect, however, that many of these are equivalent, related by various modular transformations. To establish that this is *not* the case, we can do the following exercise. In the large-N limit, the superpotential takes the form:

$$W = N(-\phi\tau^3 + 3\phi\tau^2 - 3\tau + 3)\,. \tag{2.6}$$

We can then look for supersymmetric and non-supersymmetric solutions. We have checked that there are no supersymmetric solutions, but there are non-supersymmetric ones:

$$\phi = \frac{1}{3}\left(1 + i\sqrt{2}\right) \qquad \tau = 1 + i\sqrt{2}\,. \tag{2.7}$$

The energy of these solutions is:

$$V_o = 1.5N^2\,. \tag{2.8}$$

Because the energy at the minimum of the potential is gauge invariant, and because N can be arbitrarily large, this result indicates that there are indeed an infinite number of gauge-inequivalent vacuum states in without supersymmetry in the supergravity approximation.

Note that the energy scales like N^2. This is general for states with large flux. Correspondingly, the F components of fields scale like N. This will be important shortly when we discuss the α' expansion.

These stationary points of the effective action are unlikely to have any physical significance. As we will explain in the next section, the α' expansion is invalid for them, and the vast majority are very short lived.

2.2 The α' expansion and the cutoff on the non-supersymmetric states

It is easy to see that for non-supersymmetric states, the α' expansion already breaks down for F-terms of order one. In the α' expansion, we know that there are terms with additional derivatives of the complex structure (z) fields (derivatives in the "non-compact" directions. In the effective lagrangian, these terms arise from operators in superspace with extra covariant derivatives, such as:

$$\int d^4\theta D_\alpha z D^\alpha z \bar{D}_{\dot{\alpha}} z^* \bar{D}^{\dot{\alpha}} z^*\,. \tag{2.9}$$

This includes $|F_z|^4$.

So F_z is a measure of the reliability of the α' expansion. If all other quantities are roughly order one, it is necessary the F_z be small in order to have any semblance of a controlled analysis.

2.3 The number of states at small F

At small F, one can hope to perform a self-consistent analysis. Douglas and Denef have studied the number of states at small F. Essentially this involves evaluating the integral of eqs. (2.1), (2.2), but with the delta function for unbroken supersymmetry replaced by delta functions which enforces the minimum conditions. They find that the number of non-supersymmetric states, with cosmological constant less than Λ_o and $|F| < F^*$ scales as:

$$\mathcal{N}(\Lambda < \Lambda_o, F < F^*) \approx \Lambda_o F^{*6}\,. \tag{2.10}$$

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2.3.1 A low energy understanding of the Douglas-Denef results

At first sight, the powers of F^* found by Douglas and Denef may appear surprising. But a simple effective lagrangian analysis, coupled with a plausible assumption about the distribution of couplings, yields the same results. In the case that $\Lambda_o \ll F^{*2} \ll 1$, the low energy effective theory is approximately a flat space, supersymmetric one. In this case, small supersymmetry breaking requires a light fermion to provide the longitudinal mode of the gravitino. In theories with chiral fields only, this means that there must be at least one chiral field at scales well below the fundamental scale. It is certainly more probable to have one than several, so the low energy theory consists of one chiral field (from the complex structure moduli). Call this field z, and define it so that the minimum of its potential occurs at $z = 0$ (to avoid any loss of generality, we will, below, consider a very general class of Kähler potentials). Since, by assumption, F is small, the effective lagrangian is supersymmetric, so it is described by a superpotential and a Kähler potential.

We can take the superpotential to have the form:

$$W = W_o + \alpha z + \beta z^2 + \gamma z^3 + \cdots \tag{2.11}$$

(we will see that terms higher than cubic are irrelevant not only in the renormalization group sense, but in our analysis below), and a Kähler potential:

$$K = a + bz + b^* z^* + cz^2 + c^* z^{*2} + dz^* z + \cdots . \tag{2.12}$$

In perturbation theory, $a, \ldots d \sim 1$, and we will assume that this is general.

Now we want to impose the following conditions:

1. F is small at the minimum, i.e.

$$\alpha + bW_o = F \qquad |F| < F^* . \tag{2.13}$$

2. The potential is zero at the minimum. Since F is small, this means that $W_o \sim F$.

3. The potential has its minimum at $z = 0$. Douglas and Denef provide a convenient formula for the derivatives of the potential:

$$\partial_z V = e^K (D_z D_z W \bar{D}^{\bar{z}} \bar{W} - 2 D_z W \bar{W}) . \tag{2.14}$$

4. The potential is metastable. Again, Douglas and Denef provide convenient formulas for the second derivatives of the potential. The relevant derivatives for us will be:

$$\partial_z^2 V = e^K (D_z D_z D_z W \bar{D}^{\bar{z}} \bar{W} - D_z D_z W \bar{W}) . \tag{2.15}$$

If we assume that the Kähler potential is bounded, b cannot become arbitrarily large. So from the first two conditions we learn that $\alpha \sim F$. Then the third condition gives that $\beta \sim F$. Finally, if γ is large, examining the second derivative terms, we see that the masses cannot be positive; so once more, $\gamma \sim F$.

The distribution of W_o is known in many examples to be uniform at small W_o [15]. We will argue in the next subsection that this is quite general, and also explain when one expects exceptions. So assume that the distributions of α, β and γ are uniform at small α, β, γ as complex numbers. The fraction of states with suitable α is of order $|F|^2$; similarly for β and γ. The fraction with cosmological constant smaller than Λ_o is of order Λ_o. (Throughout this discussion we are assuming that apart from the separation of the Planck scale from the supersymmetry breaking scale, there are no other large ratios of scales, i.e. the string scale and four dimensional Planck scale are comparable.) In other words, the number of states is of order

$$\int d^2\alpha d^2\beta d^2\gamma d^2 W_o \theta(\Lambda_o - V)\theta(F^* - |\alpha|)\theta(F^* - |\beta|)\theta(F^* - |\gamma|) \sim \Lambda_o F^{*6}\,. \tag{2.16}$$

These estimates appear to have a quite general character. They rely only on the assumption of roughly uniform densities in the various parameters of the low energy superpotential. In the next subsection, we explain why this assumption is quite weak. The general result accords quite closely with conventional ideas about naturalness. Phenomena on branches two and three, as we will see, are also consistent with such expectations.

2.4 Uniformity of the measure on the low energy parameters

The assumption of uniform densities about a particular point in the parameter space, $\alpha_o, \beta_o, \gamma_o, \ldots$ is quite weak. It follows if:

1. The number of states near this point is sufficiently large that the distribution is approximately continuous, i.e. there are many stationary points in this neighborhood.
2. The distribution function has a Taylor series about the point.

In our case, the assumption of uniformity is the assumption that the point where supersymmetry breaking small is not distinguished. In fact, what distinguishes branches two and three from branch one is precisely the fact that various distributions are not uniform [22], because assumption (2) does not hold. If the origin of supersymmetry breaking is dynamical, then the distribution of F's behaves as

$$\int \frac{d^2F}{|F|^2}\,. \tag{2.17}$$

In theories with unbroken supersymmetry at tree level, non-renormalization theorems account for the special character of small F. This sort of singular behavior also occurs if the hierarchy arises through warping [7]. In the third branch of [22], the W_o distribution also is singular at the origin; this is connected with the fact that the origin is a point of enhanced symmetry (R symmetry). Coupled with the requirement of small cosmological constant, it leads [22] to an even more singular distribution,

$$\Lambda_o \int \frac{d^2F}{|F|^4}\,. \tag{2.18}$$

It is these behaviors which distinguish the three branches.

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2.5 Stabilizing the Kähler moduli

The analysis of Douglas and Denef and the numerical experiments described in the previous sections ignore the Kähler moduli. Here we argue that, much as in the original KKLT analysis, for small F it is likely that the Kähler moduli are often stabilized at large values of the radii. Our effective lagrangian setup is particularly convenient for this discussion.

Following $KKLT$, we simplify the analysis by considering only a single Kähler modulus, $\rho = i\sigma + \alpha$. We suppose that the superpotential has the form:

$$W = W(z) + e^{ic\rho} \tag{2.19}$$

($W(z)$ is our earlier superpotential) while we take for the Kähler potential,

$$K = -3\ln(i(\rho - \rho^*))\,. \tag{2.20}$$

We look for a stationary point of the potential with

$$D_\rho W = \frac{\partial W}{\partial \rho} + \frac{\partial K}{\partial \rho} W\,. \tag{2.21}$$

This equation is particularly easy to analyze with the assumption that the solution lies at large σ and near $z = 0$. Then:

$$ce^{-c\sigma} \approx \frac{3}{2\sigma} W_o \tag{2.22}$$

so

$$\sigma \approx -\frac{1}{c}\ln(W_o)\,. \tag{2.23}$$

This appears to be approximately self-consistent. However, the mass of the σ field is not so small.

$$m_\sigma^2 \approx \frac{1}{\rho}|W_o|^2 \approx \rho^2 m_{3/2}^2\,. \tag{2.24}$$

The mass of the lightest z field (associated with the small F) is of order $m_{3/2}^2$. So one might worry that it is not entirely consistent to first integrate out z and then solve for ρ. The question is: at our would-be solution, how large is the tadpole for z, i.e. is the shift small. It is, in fact, easy to see that the shift is of order $1/\rho$. Roughly, the potential has the form:

$$V(z,\rho) \propto \frac{1}{\sigma^3}\left[\left|-ce^{-c\sigma} + 3\frac{W_o + \alpha z + \beta z^2 + \gamma z^3}{\sigma}\right|^2 \sigma^2 - 3\left|W_o + \alpha z + \beta z^2 + \gamma z^3 + e^{-c\sigma}\right|^2\right]. \tag{2.25}$$

As $D_\rho W$ vanishes to lowest order in $1/\sigma$, there is no contribution linear in z from the first term in the brackets, and the second term is suppressed by $1/\sigma$.

This argument suggests that plausibly all of the moduli can be fixed, with small supersymmetry breaking, in a subset of vacua.

2.6 Lifetime of non-supersymmetric states

Perhaps the best reason to suspect the existence of a physical cutoff is the question of stability. In our simple model, the infinite set of states lies largely at very large cosmological constant. As we explain here (and as is not at all surprising), most of these would-be states are extremely unstable.

To see this, recall again that the cosmological constant of these would-be states is of order N^2, i.e. it is very large. They can decay in various ways. Consider, first, decays of dS states to the region of large radius. Simple scaling arguments indicate that the action for the bounce in a semi-classical treatment scales as $1/N^2$, for large N. In other words, the semiclassical calculation breaks down already for $N = 1$; one expects that the widths of the states are order one or larger. Alternatively, one can state this in terms of supersymmetry breaking scales. For a scale F, the decay rate scales as $e^{-1/|F|^2}$. As we noted in the previous expression, once $N \gg 1$, all approximations are breaking down in the typical state; there is no clear sense in which there are metastable states at all.

The actual scaling argument is quite simple. For this we can follow Coleman in the case without gravity. Consider a potential of the form:

$$V = V_o f(\phi) \tag{2.26}$$

for some scalar field ϕ. For an $O(4)$ symmetric bounce, the action is

$$S = \int dr r^{d-1}((\partial_r \phi)^2 + V_o f(\phi)) \,. \tag{2.27}$$

Now simply rescale $r = (V_o^{-1/2})u$. to see that the action scales as $V_o^{1-d/2}$.

For the subset of states with small cosmological constant, this particular decay channel is not important, even for large supersymmetry breaking, and we have seen that there are potentially many such states.

But there are others which clearly must be considered. Typically, if we consider the space of possible fluxes, near the choice of fluxes which leads to a state of small cosmological constant, there will be AdS states. We have not worked out the theory of such decays, but it is presumably possible to decay with change of flux and emission of topological defects. As for ordinary vacuum decay, gravity surely effects these processes in important ways. Would-be decays to AdS states, for example, will sometimes not occur. In cases where they can, the final configuration is likely to be a big crunch, and the lifetimes, in general, short. Understanding these sorts of considerations is likely to bear on the question of the relative importance/likelihood of branch one.

3. Statistics on branch 2: are they computable?

Our principle observation of the last section is that one cannot calculate even the crudest statistics on branch one, the non-supersymmetric branch. This raises the question: can one actually do better on branch two (we will comment on the case of branch three in the next section).

A major point of the KKLT paper was to exhibit supersymmetric AdS vacua with all moduli fixed in regime in which the string coupling is small and the compactification radii large. In the KKLT analysis, large ρ arises when one solves the equation $D_\rho W = 0$ with $W = W_o + e^{ic\rho}$. But this form of W is only valid for large ρ, and there is no particular reason to think that there are not solutions for small ρ. For such solutions, $\langle W \rangle$ is presumably randomly distributed as a complex variable. Other quantities, like the values of couplings, might also be randomly distributed. A priori, it is not clear why these should not be at least as common as the solutions with large ρ.

This suggests that if we ask about states at small $\langle W \rangle$ and small Λ, the number with small ρ is likely comparable to and possibly significantly larger than the number with large ρ (we thank Shamit Kachru for stressing this point to us). Among these states could be states with spectra and couplings similar to those we observe.

Reasonable assumptions can be used to argue that nature might be in the large ρ regime. The observed gauge couplings are small. In the type-IIB setup, the (inverse) gauge couplings at large ρ are proportional to the value of ρ. At small ρ and strong string coupling, it seems plausible that they will be more or less uniformly distributed at small coupling. But remember we want to account for small $\langle W \rangle$ and small couplings. In the large ρ limit, small couplings are a consequence of small $\langle W \rangle$. In the small ρ region, we pay the same price for small $\langle W \rangle$, and we pay an additional price (10^{-6} perhaps?) for small couplings. Small couplings might well be selected by anthropic considerations, or they may be a piece of data we wish to impose.

If we take coupling unification seriously, something like large ρ looks even more plausible. We will not propose here a mechanism to understand even tree level unification of couplings, but if there is some important class of states for which unification is common, it is surely more likely to occur in the large ρ region (or its analog), then randomly in the strong coupling region.

4. The $W = 0$ branch: discrete R symmetries

In [22], it was argued that on the $W = 0$ branch, small cosmological constant favors very low scale breaking. Roughly, the distribution behaves as:

$$\mathcal{N}(|F| < F^*, \Lambda < \Lambda_o) \propto \Lambda_o \frac{1}{F^*}. \tag{4.1}$$

So, while on the one hand, one might expect the fraction of states with unbroken R symmetry might be small, there is a big gain when one selects for the cosmological constant. Further gains might arise from other selection effects, such as proton decay.

Vanishing W might arise by accident, or as a result of symmetries. Ref. [23] performed some counting in simple models. As one might naively guess, in most cases, these authors found vanishing W results from discrete R symmetries. It is not hard to see that such a symmetry is likely to yield both unbroken supersymmetry and vanishing W. Suppose that one has a discrete R symmetry under which W transforms by a phase α. Suppose that there are some number of fields, Z_i, $i = 1, \ldots, n$, which also transform by α, and some number, ϕ_A, $A = 1, \ldots, m$, which are neutral. Then the superpotential has the form:

$$W = \sum_{i=1}^{n} Z_i f_i(\{\phi_A\}) \,. \tag{4.2}$$

Then provided $m > n$, there will be supersymmetric solutions with $W = 0$. There will not be supersymmetric solutions if $m < n$. One can readily modify this argument in the case that there are various fields which transform with some other power of α.

In the landscape, it is easy to see that states with $m > n$ are statistically significantly favored. In the Calabi-Yau case, there is a pairing of complex structure moduli and fluxes. Loosely, in order that the low energy theory respect the symmetry, we expect that for any complex structure modulus which transforms under the symmetry, we must set the corresponding flux to zero. So in order to have a large number of states, and a small number of vanishing fluxes, we must have a small number of fields which transform under the symmetry. So we are in the limit $m > n$, above. Of course, without a complete survey, we cannot make a sweeping statement, but it seems likely that R symmetries in the landscape will lead typically to unbroken supersymmetry and vanishing W.

It is helpful to illustrate these considerations with some examples. One case where it is possible to be very explicit is that of IIB theory on a T_6/Z_2 orientifold. Focus on the point in the moduli space where the T_6 is a product of three two-tori of equal size. In this case, before turning on fluxes, the theory has a variety of R-symmetries. There are Z_4 symmetries which rotate each of the separate planes; there is also an S_3 symmetry which permutes the planes. We can attempt to turn on only fluxes which respect the Z_4 symmetry in the first plane. Starting from the list in [25] we can make a complete list: From the subset:

$$\alpha_{ij} = \frac{1}{2}\epsilon_{ilm} dx^\ell \wedge dx^m \wedge dy^j \qquad \beta_{ij} = -\frac{1}{2}\epsilon_{jlm} dy^\ell \wedge dy^m \wedge dy^i \tag{4.3}$$

α_{12},α_{13}, α_{21}, α_{31} and similarly for the components of β. For simplicity, take for the RR and NS three-forms, F and H:

$$\frac{1}{(2\pi)^2\alpha'} F_{(3)} = a^{12}\alpha_{12} + b_{12}\beta^{12} \qquad \frac{1}{(2\pi)^2\alpha'} H_{(3)} = c^{12}\alpha_{12} + d_{12}\beta^{12} \,. \tag{4.4}$$

Correspondingly, the superpotential is:

$$(a^{12} - \phi c^{12}) \mathrm{cof} \tau_{12} + (b_{12} - \phi d_{12})\tau^{12} \,. \tag{4.5}$$

We look for a solution of the form

$$\tau = \begin{pmatrix} i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & i \end{pmatrix} . \tag{4.6}$$

Then $W = 0$, and one finds for ϕ:

$$\phi = \frac{ia^{12} - b_{12}}{ic^{12} - d_{12}} \,. \tag{4.7}$$

Two remarks about this solution are important. First, there is a multiparameter set of solutions. Many of the elements of τ are undetermined. Second, we should also impose the tadpole cancellation condition. In this case, this reads:

$$-a^{12}d_{12} + b_{12}c^{12} = L \tag{4.8}$$

with $L \leq 16$ (and integer). Restricting ϕ to the fundamental region leaves a limited set of fluxes. Note, in particular, that if $L = 1$, ϕ is an $\mathrm{SL}(2, \mathbb{Z})$ transform of i. There are hundreds of such solutions. Additional solutions are obtained by permuting the planes, and considering points with other symmetries, such as Z_6. This model is not realistic at many levels. The vacua which we have enumerated here, as pointed out in [25] have more than four supersymmetries, for example. Still, they exhibit many of the features we discussed above.

In these cases, the superpotential transforms with a phase, $\alpha = e^{\frac{2\pi i}{4}}$; the fields τ^{12} and τ^{13} also transform with phase α. The only other field which transforms non-trivially is τ^{11}. Because the superpotential in this model is at most cubic, τ^{11} cannot appear and is thus not determined. The rest of the superpotential has the structure:

$$W = \tau^{12} f(\tau^{22}, \tau^{23}, \tau^{33}, \phi) + \tau^{13} g(\tau^{22}, \tau^{23}, \tau^{33}, \phi)\,. \tag{4.9}$$

With sufficiently general choices of fluxes, one easily breaks all of the R symmetries. Still, in this model, one often finds $W = 0$ [23]. This may be a feature of this class of superpotentials, which are rather restricted polynomials, but it might also be more general. This is an important question to investigate.

Now consider the case of a more complicated Calabi-Yau space. The quintic in CP^4 is well studied, and its symmetries are explained in textbooks [26]. In particular, there is a highly symmetric point with symmetry Z_5^4 (times a permutation symmetry, which we will ignore). Calling the coordinates of CP^4 Z_i, $i = 1, \cdots, 5$, the quintic polynomial equation which defines the space is:

$$\sum Z_i^5 = 0\,. \tag{4.10}$$

This is invariant under the separate symmetries $Z_i \to \alpha Z_i$, where α is now a fifth root of unity. The complex structure moduli are in one to one correspondence with quintic polynomials. Now we can require that the fluxes preserve just one of the Z_5's, say that acting on the first coordinate. Then the fluxes which are paired with polynomials (30 in all) which transform under this Z_5 must vanish; examples include $Z_1^3 Z_2^2$, $Z_1^3 Z_2 Z_3$, and so on. So, in fact, there are 46 Z-type fields, in our previous notation, and 55 ϕ type fields. One can try and preserve more R symmetries. Then many more fluxes must vanish — and there are many fewer states. With a sufficiently large symmetry, there will only be supersymmetry breaking solutions.

In this example, the dimensionality of the flux space is reduced by more than 1/3. While this is a dramatic reduction, there may be other selection effects which favor such states. The cosmological constant itself is one; compared to the non-supersymmetric case, this could well be a factor of 10^{50} or more; compared to branch two, it could easily be 10^{25}.

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Proton decay is an additional issue; without symmetries, on branch two, anthropic reasoning alone cannot explain the value of the proton lifetime, and the anthropic constraint itself requires a huge suppression (10^{10} in several couplings). There may be other important selection effects as well. How general this rather drastic reduction in the dimensionality of the space — and, as a result, in the number of states — is a straightforward problem worthy of further study.

Finally, we need to ask about those moduli which are unfixed at the level of the superpotential equations. These will be fixed by the dynamics which breaks supersymmetry. In general, there is no reason that these effects should be calculable. There are, potentially, severe cosmological problems associated with these fields. At the same time, this is a framework in which axions might well arise, and interesting, testable long range forces (we thank Savas Dimopoulos for stressing this possible positive aspect of these fields to us). We will comment further on these possibilities in the next section.

5. Prospects for predictions

Of the three branches of the flux landscape which we have distinguished, the first, non-supersymmetric branch is the most problematic. We have seen that, thanks to the work of Douglas and Denef [15], the existence of this branch is as well established as any branch of the landscape (explicit constructions of states on branch one appear in [27]). We have given a simple argument based on the assumption of uniform distribution of couplings in the low energy effective action which reproduces their results. But the same arguments which establish the statistics of these states for small F demonstrate that the vast majority of states in this branch are inaccessible. While within the set of states which can be reliably studied, the number of supersymmetric and non-supersymmetric states are comparable, we can only conjecture whether there are or are not far more non-supersymmetric states than supersymmetric ones, or, for that matter, whether the number of states on this branch is finite. Even if we ignore cosmological issues and simply assume that we should compare the relative numbers of states, we cannot decide whether or not this branch dominates. We have indicated some simple cosmological questions which might affect this question.

If nature does lie on this branch, the situation is very disappointing. The problem is not that one does not predict low energy supersymmetry, but that one is very unlikely to predict anything. Because of the lack of control, there is no underlying small parameter and approximation scheme which will allow one to extract correlations of any kind. The parameters of low energy physics then presumably arise anthropically or randomly.

While one is largely in the dark about branch one, an optimist might proceed by studying statistics at small F, hoping that some features will persist into the inaccessible dominant region. One question which one can address is how hierarchies arise. The work of [16, 28] indicates that warping is common in the supersymmetric vacua. This analysis remains valid in the almost supersymmetric case, when there are many moduli. This feature might survive into the strong coupling regime. An alternative origin for hierarchies could be conventional dynamical supersymmetry breaking, as in technicolor theories. Just as in

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the case of branch two, one expects that chiral gauge theories are common, and associated with these dynamical symmetry breaking and hierarchical scales. Of course, problems of precision electroweak physics, flavor and the like will all require some resolution. And the strong coupling problem means that it is probably impossible to say more than what effective field theorists have argued for many years.

There are some phenomenological reasons to think we might not be on branch one. As we have just said, it would be necessary to understand why solutions of the usual problems of technicolor and/or warped spaces are solved in some generic fashion. More generally, in this strongly coupled regime, the parameters of low energy physics would seem to be either random or anthropic. While it is plausible that several of the parameters of the Standard Model are anthropic, it is implausible that most of them are. There are clear patterns and regularities. The small value of $\theta_{\rm qcd}$ is one example [17, 29]. The heavy quark and lepton masses and mixings do not appear random, yet they are probably not anthropic; the neutrino masses are much larger than one might expect to arise randomly in such a picture, and again there is no obvious anthropic explanation. (Of course, these may be arguments that not only are we not on branch one but that the landscape itself is not correct.)

One example of a prediction which is *not* likely to arise on branch one is split supersymmetry with very large scale of supersymmetry breaking [30]. Small masses for all the gauginos are probably not required anthropically. One needs broken supersymmetry at the high scale, but an unbroken R symmetry. But we have given simple arguments that in the landscape, unbroken R symmetry is necessarily associated with unbroken supersymmetry — in fact, that this puts one on branch three. A form of split supersymmetry with a scale of order 10's of TeV's might arise for other reasons [22]. But while the phenomenology of much larger breakings is interesting to to explore, it it is not a generic feature of states of the landscape.

On branches two and three, it is more likely that one can make real predictions, and resolve some of the puzzles of the Standard Model. The number of states appears to be finite, and the statistics seem compatible with conventional notions of naturalness. In [22], some features of branch two and three were noted. Here we looked mainly at the relative populations of branches two and three. We adopted the point of view that vanishing W arises, in general, as a result of discrete R symmetries. This viewpoint finds some support in the work of [23], who, while finding some cases where vanishing W arises by accident, largely found that the explanation lies in such symmetries. We explained why discrete R symmetries are likely to lead to both unbroken supersymmetry and unbroken R invariance. We also explained why some moduli are likely to be unfixed in these models, before non-perturbative effects, and particularly supersymmetry breaking, are taken into account. This is potentially problematic for cosmology, but also could be a striking prediction[1]

We asked the fraction of states which have vanishing W at tree level in a particular example with a large number of moduli. In one example, we found that requiring an R symmetry reduced the dimension of the moduli space by about 1/3. This is presumably an

[1]We thank Savas Dimopoulos for stressing this aspect of light moduli to us.

enormous price in landscape terms, quite possibly larger than the gain (about 10^{40}) which one might imagine comes from the ease of obtaining a small cosmological constant. On the other hand, discrete symmetries might be required by other considerations as well: proton stability and dark matter. These could easily introduce additional compensating factors of 10^{40} or so.

Making further progress on the phenomenology of branches two and three requires more knowledge of the distribution of gauge groups and chiral matter content; some preliminary studies of these questions have been reported [31 – 33]. More information about statistics of discrete symmetries is also essential. It would seem straightforward to explore these questions. With this sort of data, we could, for example, assess the likelihood of low energy dynamical supersymmetry breaking in branches two and three. As explained in [22], if gauge singlets are not common, on branch two one is lead to a prediction which in fact coincides with one version of split supersymmetry. On branch three, there are a rich set of questions one might hope to address. We have begun some preliminary studies of these statistics.

Finally, the observations of this paper suggest a more refined (perhaps we should say mature) view of naturalness. In the landscape, conventional ideas of naturalness often have a sharp realization. Our success in reproducing the formulas of Douglas and Denef is an example. Enumerating the terms in a low energy effective action, and assuming uniform distributions for them, yields the results of DD's more sophisticated microscopic calculation. Our arguments about physics in the different branches, and about the relevant populations of branch two and three are similar. But we are presently stymied in our efforts to decide about the population of, and importance of, branch one relative to branches two and three. Taking the cutoff in the DD calculation to be order one makes it plausible that branch one and two have comparable populations; the work of Silverstein and collaborators [34] and of Bobkov [35], suggesting that there are numerous other constructions of non-susy vacua, might suggest that the population of branch one is huge compared to that of branch two. It is also possible that cosmological considerations will ultimately force us to phrase the question in some very different way.

From all of this, it appears that it is difficult, in principle, to decide whether or not the landscape predicts supersymmetry. If one hypothesizes that it does, than one may be able to make predictions about the scale and nature of supersymmetry breaking. If it does not, it will be difficult to extract any real predictions; at best, one might give a rationale for the appearance of technicolor or warping in low energy physics — or their absence.

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References

[1] R. Bousso and J. Polchinski, *Quantization of four-form fluxes and dynamical neutralization of the cosmological constant*, *JHEP* **06** (2000) 006 [hep-th/0004134].

[2] J.L. Feng, J. March-Russell, S. Sethi and F. Wilczek, *Saltatory relaxation of the cosmological constant*, *Nucl. Phys.* **B 602** (2001) 307 [hep-th/0005276].

[3] K. Becker and M. Becker, *M-theory on eight-manifolds*, *Nucl. Phys.* **B 477** (1996) 155 [hep-th/9605053].

[4] J. de Boer, R. Dijkgraaf, K. Hori, A. Keurentjes, J. Morgan, D. R. Morrison and S. Sethi, *Triples, fluxes and strings*, *Adv. Theor. Math. Phys.* **4** (2002) 995 [hep-th/0103170].

[5] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097].

[6] B.S. Acharya, *A moduli fixing mechanism in M-theory*, hep-th/0212294; *Compactification with flux and Yukawa hierarchies*, hep-th/0303234.

[7] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *de Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240].

[8] L. Susskind, *The anthropic landscape of string theory*, hep-th/0302219.

[9] S. Weinberg, *The cosmological constant problem*, *Rev. Mod. Phys.* **61** (1989) 1.

[10] J. Garriga and A. Vilenkin, *On likely values of the cosmological constant*, *Phys. Rev.* **D 61** (2000) 083502 [astro-ph/9908115].

[11] S. Weinberg, *The cosmological constant problems*, astro-ph/0005265.

[12] M.R. Douglas, *The statistics of string/M-theory vacua*, *JHEP* **05** (2003) 046 [hep-th/0303194].

[13] S. Ashok and M.R. Douglas, *Counting flux vacua*, *JHEP* **01** (2004) 060 [hep-th/0307049].

[14] M.R. Douglas, *Statistics of string vacua*, hep-ph/0401004.

[15] F. Denef and M.R. Douglas, *Distributions of flux vacua*, *JHEP* **05** (2004) 072 [hep-th/0404116].

[16] F. Denef and M.R. Douglas, *Distributions of nonsupersymmetric flux vacua*, *JHEP* **03** (2005) 061 [hep-th/0411183].

[17] T. Banks, M. Dine and E. Gorbatov, *Is there a string theory landscape?*, *JHEP* **08** (2004) 058 [hep-th/0309170].

[18] T. Banks, *Landskepticism or why effective potentials don't count string models*, hep-th/0412129.

[19] B. Freivogel and L. Susskind, *A framework for the landscape*, *Phys. Rev.* **D 70** (2004) 126007 [hep-th/0408133].

[20] M.R. Douglas, *Statistical analysis of the supersymmetry breaking scale*, hep-th/0405279;

[21] L. Susskind, *Supersymmetry breaking in the anthropic landscape*, hep-th/0405189.

[22] M. Dine, E. Gorbatov and S. Thomas, *Low energy supersymmetry from the landscape*, hep-th/0407043.

[23] O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, *Enumerating flux vacua with enhanced symmetries*, *JHEP* **02** (2005) 037 [hep-th/0411061].

[24] K.R. Dienes, E. Dudas and T. Gherghetta, *A calculable toy model of the landscape*, hep-th/0412185.

[25] S. Kachru, M.B. Schulz and S. Trivedi, *Moduli stabilization from fluxes in a simple IIB orientifold*, *JHEP* **10** (2003) 007 [hep-th/0201028].

[26] M.B. Green, J.H. Schwarz and E. Witten, *Superstring theory*, Cambridge University Press, Cambridge 1987.

[27] A. Saltman and E. Silverstein, *The scaling of the no-scale potential and de Sitter model building*, *JHEP* **11** (2004) 066 [hep-th/0402135].

[28] A. Giryavets, S. Kachru and P.K. Tripathy, *On the taxonomy of flux vacua*, *JHEP* **08** (2004) 002 [hep-th/0404243].

[29] J.F. Donoghue, *Dynamics of M-theory vacua*, *Phys. Rev.* **D 69** (2004) 106012 [hep-th/0310203], erratum *ibid.* **D 69** (2004) 129901.

[30] N. Arkani-Hamed and S. Dimopoulos, *Supersymmetric unification without low energy supersymmetry and signatures for fine-tuning at the LHC*, hep-th/0405159.

[31] J. Kumar and J.D. Wells, *Landscape cartography: a coarse survey of gauge group rank and stabilization of the proton*, *Phys. Rev.* **D 71** (2005) 026009 [hep-th/0409218].

[32] J.P. Conlon and F. Quevedo, *On the explicit construction and statistics of Calabi-Yau flux vacua*, *JHEP* **10** (2004) 039 [hep-th/0409215].

[33] R. Blumenhagen, F. Gmeiner, G. Honecker, D. Lüst and T. Weigand, *The statistics of supersymmetric D-brane models*, *Nucl. Phys.* **B 713** (2005) 83 [hep-th/0411173].

[34] A. Saltman and E. Silverstein, *A new handle on de Sitter compactifications*, hep-th/0411271.

[35] K. Bobkov, *Volume stabilization via α' corrections in type-IIB theory with fluxes*, *JHEP* **05** (2005) 010 [hep-th/0412239].